STUDIES ON LINEAR DELTA MODULATION AND PREDICTIVE QUANTIZATION SYSTEMS

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CERTIFICATE

This is to certify that the work on 'STUDIES ON LINEAR DELTA MODULATION AND PREDICTIVE QUANTIZATION SYSTEMS' by Major Vijay Raheja has been carried out under my supervision and this has not been submitted elsewhere for a degree.

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TABLE OF CONTENTS

ABSTRACT

A Linear Delta Modulation system has been simulated. The signals used as input are a flat band-limited Gaussian signal and a RC-shaped Gaussian signal. Thereafter a Differential Pulse Code Modulation system has been simulated. Herein Predictive Quantization systems using linear predictors have been studied with multi-bit quantizers having 4,8 and 16 levels. A predictive system with a Gauss-Markov input has also been studied. The signal to noise ratios obtained are best for a Gauss-Markov input. Finally an attempt has been made at bit rate reduction of the transmitted signal.

Results obtained compare favourably with results obtained earlier. However, the results for bit rate reduction do not give any scope for incorporating in a practical system.

CHAPTER 1

INTRODUCTION

In a communication system the effect of interference in the transmission path can be reduced considerably by quantizing the signal into a finite number of levels, assigning a code to each of these levels and then transmitting these code words in the form of a two level pulse stream. This is the well known n bit Pulse Code Modulation (PCM) system. Even if a noiseless transmission is assumed the performance of such systems is limited mainly due to error added to the signal on reception. This error manifests itself in the form of what is known as quantization noise. For an acceptable performance 7 or 8 bit PCM is necessary for speech and television transmission. This naturally constrains the number of signals that can be transmitted through a given channel.

The signal to noise ratio for a PCM system is given by [9]

$$\mathrm{SNR(dB)} = 6n - 7.2,$$

where n is the number of bits being used for coding. The above formula assumes that the quantizer is aligned such that the maximum range of the input signal is between -4σ and $+4\sigma$ where σ^2 is the input variance of the signal. It has been seen that for a zero mean Gaussian input signal

samples will fall outside this range with a probability less than 10^{-4}.

Speech sampled at Nyquist rate exhibits a significant correlation C_1, between adjacent samples. As a consequence of this the variance of the difference between two consecutive samples is less than the variance of the speech signal itself. Thus we have

$$E(e_t^2) = E[(X_t - X_{t-1})^2]$$

$$= E[X_t^2] + E[X_{t-1}^2] - 2E[X_t X_{t-1}]$$

$$= E[X_t^2]\,[2(1 - C_1)],$$

where X_t and X_{t-1} represent the samples at time t and (t-1) respectively, e_t represents the difference signal.

Now if C_1 is greater than 0.5 e_t has a smaller variance than X. It is therefore simpler to quantize e_t instead of X. An integrator is used to reconstruct X from quantized values of e_t. This technique of quantizing the difference signal is called Differential Pulse Code Modulation (DPCM). This technique permits employment of a smaller value of n and thereby reduces bandwidth. Considering a subjective evaluation it is found that for the same bit rate DPCM is superior to PCM.

Oversampling a signal increases the adjacent sample correlation and we can have a relatively simple technique of quantization which uses 1 bit. This 1 bit version of DPCM is called Delta Modulation (DM).

Figure 1(a) depicts a DM system schematically. The bandlimited input signal is sampled at a rate f_s which is much higher than the Nyquist rate. At every sample, the sign b_r of the difference between X_t and the latest approximation to it Y_{t-1} is generated, this is multiplied by a step Δ in the direction of b_t. Finally the high frequency waveform Y is low pass filtered to the original signal bandwidth.

The basic equations characterizing DM are given as under

$$b_t = \mathrm{Sgn}(X_t - Y_{t-1})$$

$$Y_t = Y_{t-1} + \Delta \cdot b_t$$

2. DELTA MODULATION

The block schematic of a Delta Modulation codec is shown in Fig. 1(b). The waveform p(t) at the output of the delta modulator consists of pulses of duration τ seconds placed T seconds apart ($T \gg \tau$) having amplitude $\pm\Delta$ volts. These pulses occur at clock times $f_s = \frac{1}{T}$ where f_s is considerably greater than the Nyquist rate.

Figure 1(c) depicts the output waveform of a delta modulator. The difference between X(t) and Y(t) where Y(t)

is the reconstructed signal constitutes the error signal e(t). This error signal is quantized to limits $\pm\Delta$ volts depending upon its sign. The output of the quantizer is sampled every T seconds to produce L(t) pulses.

If e(t) is greater than or equal to zero at a clock instant a positive pulse will be produced at the output of the encoder. When Y(t) becomes greater than X(t), e(t) will become less than zero and a negative pulse will thus emanate from the encoder. In short the encoder attempts to reduce the error wave form when an input signal is present by producing positive and negative pulses.

Figure 1(c) also illustrates the two types of quantizing errors in Delta Modulation - slope overload distortion and granular noise. Slope overload is said to occur when the step size Δ is too small to follow a steep segment of the input waveform. Granularity refers to the situation where the staircase function Y(t) hunts around a relatively flat segment of the input function X(t), with a step size that is too large relative to the local slope characteristics of the input.

The decoder consists of an integrator and a filter. Assuming additive channel noise to be zero the L(t) is recovered and integrated to yield Y(t) which is identical to the feedback to the error point in the encoder. As this

Y(t) differs from the input signal X(t) by a relatively small error signal e(t), the output of the integrator in the decoder is thus a good reproduction of the original signal X(t), and has some quantization noise added to it. This can be reduced by either increasing the sampling frequency or using a multi level quantizer instead of the two level quantizer described herein. The step like nature of Y(t) is removed by passing the signal through a filter whose pass band is the same as message band. When no input is applied to the encoder it is said to be in an idling state and the output of the delta modulator is 101010 ....

The effect of additive channel noise is to introduce accumulative error in the demodulated signal. This is due to the integrator in the feedback path and can be reduced by putting a leaky integrator in the feedback path. This will however limit the slope and amplitude. A better approximation of the input signal is possible by using a double integration in the feedback path. The output wave form is smoother than in the case of a single integrator. Low pass filtering further smoothens the signal and it is a better reproduction of the input signal. The disadvantage of a double integrator is that it is slow to follow fast changes of the slope of the message signal.

Consider a sinusoidal input given by

$$X(t) = E_m \operatorname{Sin} 2\pi f_m t \tag{1.1}$$

$$X'(t) = E_m 2\pi f_m \operatorname{Cos} 2\pi f_m t \tag{1.2}$$

$$\text{Max slope} = E_m 2\pi f_m, \tag{1.3}$$

for a step length K and a sampling frequency f_s we have

$$\text{Slope of } Y(t) = Kf_s \tag{1.4}$$

Thus for no slope overload

$$E_m 2\pi f_m \ll Kf_s \tag{1.5}$$

Let E_{mm} represent the maximum amplitude which does not overload.

$$\text{Therefore } E_{mm} = Kf_s/2\pi f_m \tag{1.6}$$

In the idling state the wave form $Y(t)$ at the output of the integrator is square with peak to peak value of K. To disturb idling input sinusoid must have an amplitude $E_m \gg K/2$. This thus gives us the dynamic range (DR) of the systems as under

$$DR = \frac{f_s}{\pi f_m} \tag{1.7}$$

The quantization noise is due to the error signal e(t). Assuming that the spectral density $S_n(f)$ of this error signal is flat over the message band we have quantization noise N_q related by the relation

$$N_q^2 = K_q \frac{f_c K^2}{f_s} , \tag{1.8}$$

where K_q is a constant of proportionality and is of the order of 1/3 and f_c is the cut off frequency of the low pass filter.

The signal to quantization noise ratio is defined as the input signal power (S^2) to the decoded noise power N_q^2. For a sinusoid the maximum input power is given by

$$S^2 = \frac{E_{mm}^2}{2}$$

$$= \frac{1}{2} \left[\frac{Kf_s}{2\pi f_m}\right]^2 \tag{1.9}$$

combining 1.8 and 1.9 we have signal to quantization noise ratio given by

$$SNR = \frac{1}{2K_q} \left[\frac{f_s}{2\pi f_m}\right]^2 \times \frac{f_s}{f_c}$$

$$= \frac{f_s^3}{8K_q \pi^2 f_m^2 f_c} \tag{1.10}$$

For a double integrator in the feedback path the signal to noise ratio is given by

$$SNR = \frac{f_s^5}{8\pi^2 K_q f_m^2 f_c^3} \tag{1.11}$$

The increase in SNR is 9 dB and 15 dB per octave in the case of the single integrator and double integrator respectively.

3. DIFFERENTIAL PULSE CODE MODULATION

This technique as brought out earlier emanates from PCM and differs to the extent that instead of quantizing the original signal we quantize a difference signal which is generated from the original signal and an estimate of this signal. The estimate may be generated by incorporating a predictor in the feedback path of a DPCM system. Such systems will be referred to as the Predictive Quantizing systems.

3.1 Predictive Quantizing Systems

A schematic diagram of a predictive quantizing system is shown in Fig. 2. The difference between the input signal and the predicted signal, which is generated from the past history of the signal, is fed as input to a multi-level quantizer. The predictor used in the feedback path may be a linear or a non-linear type. The theory of linear predictors is discussed in the succeeding paragraphs.

3.2 Linear Prediction

Consider a stationary signal $x(t)$ with zero mean and variance σ^2 being sampled at instants $t_1, t_2, \ldots\ t_n, \ldots$ The sample values at these instants are given by $x_1, x_2, \ldots$ $x_n, \ldots$

The linear estimate of a sample x_0 at the next sampling instant is given by

$$\hat{x}_0 = a_1x_1 + a_2x_2 + \ldots, + a_nx_n,$$

where a's are real numbers. The difference or error signal is given by

$$e_0 = x_0 - \hat{x}_0$$

A block diagram showing such a predictor is given in Fig. 3(a). The D's represent the delays--D_1 implying a unit delay, D_2 two delays and D_n indicating n delays.

The best estimate of x_0 will be that value of $\hat{x}_0$ which gives minimum expected value of the squared error. To determine the values of a's which satisfy this condition we take partial derivative of $E[(x_0 - \hat{x}_0)^2]$ with respect to each one of the a's and set it equal to zero.

$$\frac{\partial E[(x_0 - \hat{x}_0)^2]}{\partial a_i} = \frac{\partial E[(x_0 - (a_1x_1 + a_2x_2 + \ldots + a_nx_n))^2]}{\partial a_i}$$

$$= -2E[x_0x_i - (a_1x_1 + a_2x_2 + \ldots + a_nx_n)x_i]$$

$$i = 1, 2, \ldots n$$

Equating this to zero to get a minimum we have

$$E[(x_0 - (a_1x_1 + a_2x_2 + \ldots + a_nx_n))x_i] = 0 \qquad (1.12)$$

$$E[(x_o - \hat{x}_0)x_i] = 0 \qquad (1.13)$$

$$i = 1, 2, \ldots\ldots, n.$$

Let covariance of x_i and x_j be given by

$$R_{ij} = E(x_i\ x_j) \qquad (1.14)$$

Thus rewriting (1.13) we get the best linear mean square estimate as

$$R_{oi} = a_1R_{1i} + a_2\ R_{2i} + \ldots + a_n\ R_{ni} \qquad (1.15)$$

$$i = 1, 2, \ldots\ldots n$$

This gives rise to a set of n simultaneous equations having n unknowns in a_i, $i = 1, 2, \ldots\ldots n$, which can be determined if the covariances R_{ij} are known. Rewriting (1.15) in matrix form we have

$$\begin{bmatrix} R_{01} \\ R_{02} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ R_{0n} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{21} & \cdots\cdots & R_{n1} \\ R_{12} & R_{22} & \cdots\cdots & R_{n2} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ R_{1n} & R_{2n} & \cdots\cdots & R_{nn} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} \qquad (1.16)$$

Considering the fact that x(t) is a stationary process we have

$$R_{ij} = R_{ji}$$

$$R_{(ij)} = R_{(i+k(j+k))}, \quad k = 0,1,2, \ldots\ldots n \qquad (1.17)$$

Hence (1.16) can be rewritten as under

$$\begin{bmatrix} R_{01} \\ R_{02} \\ \vdots \\ \vdots \\ R_{0n} \end{bmatrix} = \begin{bmatrix} \sigma^2 & R_{01} & R_{02} & R_{03} \cdots\cdots & R_{0n-1} \\ R_{01} & \sigma^2 & R_{01} & R_{02} \cdots\cdots & R_{0n-2} \\ \vdots & & & & \\ \vdots & & & & \\ R_{0n-1} & R_{0n-2} & \cdots\cdots\cdots\cdots & & \sigma^2 \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix} \qquad (1.18)$$

We can rewrite (1.18) in notational form as

$$A_{opt} = \Gamma^{-1} \Sigma$$

where

$$\Gamma = \begin{bmatrix} \sigma^2 & R_{01} & R_{02} & \cdots\cdots & R_{0n-1} \\ R_{01} & \sigma^2 & R_{01} & \cdots\cdots & R_{0n-2} \\ \vdots & & & & \vdots \\ R_{0n-1} & R_{0n-2} & \cdots\cdots\cdots\cdots & & \sigma^2 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} R_{01} \\ R_{02} \\ \cdot \\ \cdot \\ \cdot \\ R_{0n} \end{bmatrix}$$

Thus by knowing the covariances we can determine the unknowns a_i's, $i = 1, 2, \ldots\ldots n$.

A predictive quantizing system is shown in Fig. 3(b). The sampler converts the analogue input $x(t)$ into its digital form. Let x_i be the output sequence of the sampler at the ith instant. The output of the predictor at this instant is $\hat{x}_i$ and is given by the sum of $\hat{x}_{i-1}$ and the (i-1)th output of the quantizer. The error e_i which constitutes the input to the quantizer is given by the difference between x_i and $\hat{x}_i$. The quantizer has N discrete levels. The output of the quantizer is thus the error signal e_i and some quantization noise q_i. On the receive end the input is $(e_i + q_i)$. The predictor as on the transmitter end forms a part of the feed-back loop and produces the linear estimate $\hat{x}_i$, which is mixed with the input signal to yield $(x_i + q_i)$. This is then passed through a low pass filter to yield $y(t)$ which is a replica of $x(t)$ with some quantization noise added.

The signal to quantization noise ratio for such a system is obtained from the following [7] :

$$\mathrm{SNR(dB)} = -4.35 + 6n + 10 \log \frac{\sigma^2}{\sigma_e^2} ,$$

where n represents the discrete levels that the quantizer takes and σ_e^2 is the mean square value of the error signal.

4. ADAPTIVE DELTA MODULATION

A more flexible means of matching quantizer step size to input signal variance is the use of step size adaption based on quantizer memory. This involves modifying the step size by a factor depending on the knowledge of the previous quantizer output. In its simplest form the scheme operates with a one bit memory. A simple algorithm using such a one bit memory for automatic step size correction is as follows [6]

$$|K_i| = M_1 \times |K_{i-1}| \quad \text{if} \quad b_i = b_{i-1}$$

$$|K_i| = M_2 \times |K_{i-1}| \quad \text{if} \quad b_i \neq b_{i-1}$$

where $M_1 \geqslant 1$ and $M_2 \leqslant 1$ and b_i is the ith output of the quantizer. In other words in case the ith bit output of the quantizer as given by b_i is the same as the (i-1)th bit the magnitude of the step size k_i increases by a factor M_1. In contrast the step size decreases by a factor M_2 in case b_i and b_{i-1} are different.

Adaptive Delta Modulation systems have not been considered during the course of this study.

5. SCOPE OF WORK

In the work reported here digital computer studies of Linear Delta Modulation and Predictive Quantization systems have been made. Specifically the following simulation studies have been carried out :

(a) Simulation of a DM system with a band limited Gaussian signal as input having :

  (i) Uniform power spectrum,

  (ii) RC integrated spectrum.

(b) Simulation of a DM system with a Gauss-Markov signal as input.

(c) Simulation of Predictive Quantizing systems using 2,4,8 and 16 levels of quantization.

(d) An attempt has been made at reducing the transmitted bit rates by not transmitting the encoder output bit stream directly but by coding it further in another encoder.

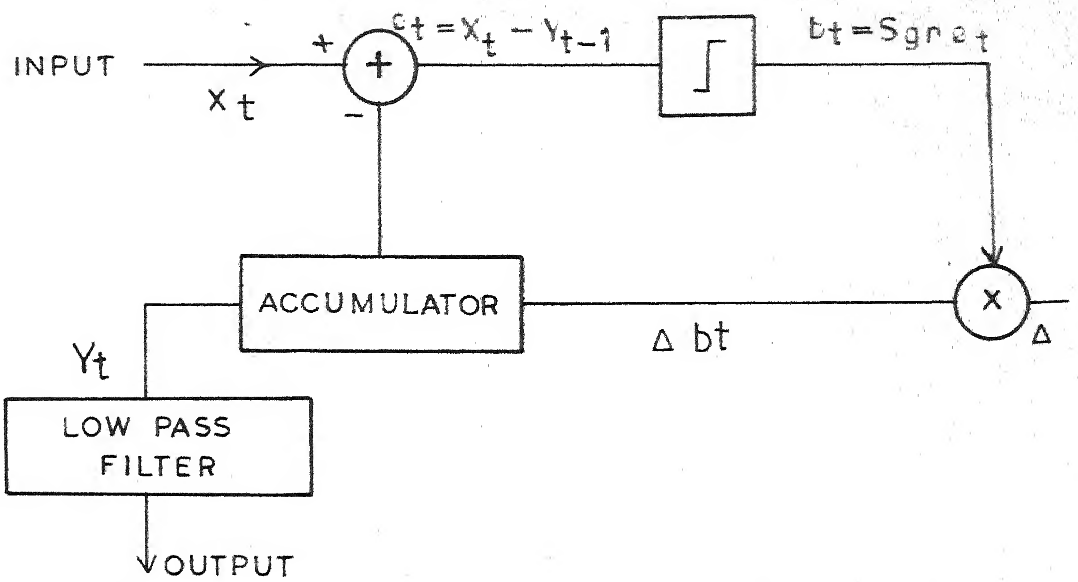


FIG.1(a) BLOCK DIAGRAM OF A DM SYSTEM

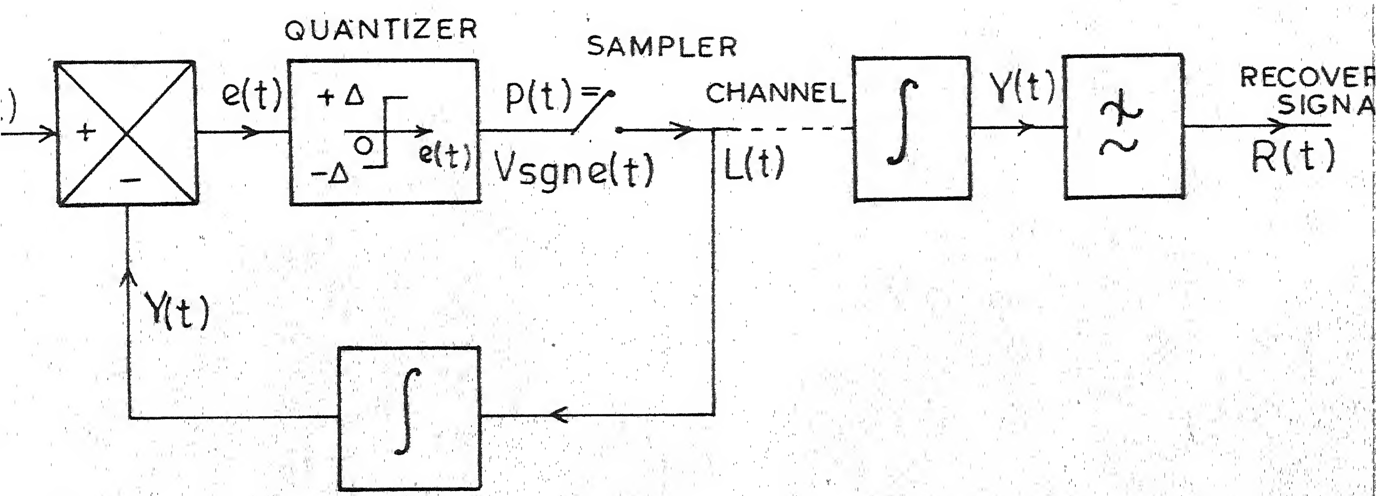


FIG.1(b) DELTA MODULATOR SYSTEM

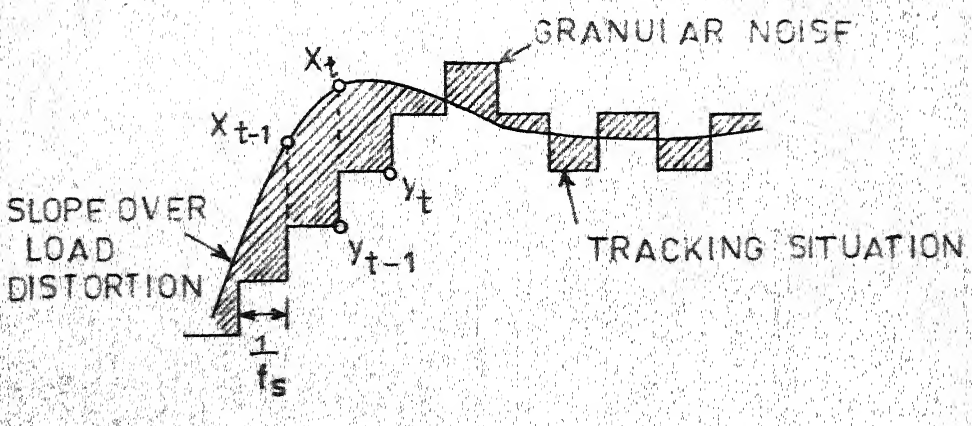


FIG.1(c) DELTA MODULATOR OUTPUT WAVEFORM

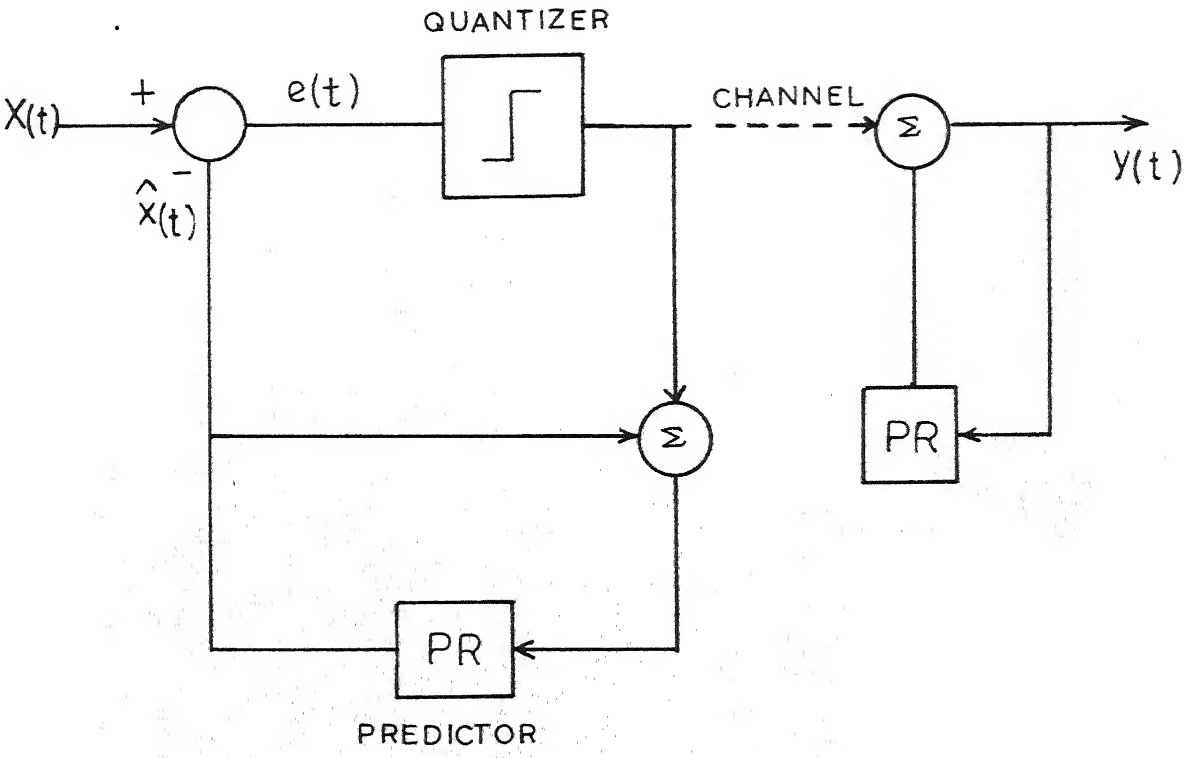


FIG.2. DIFFERENTIAL PCM

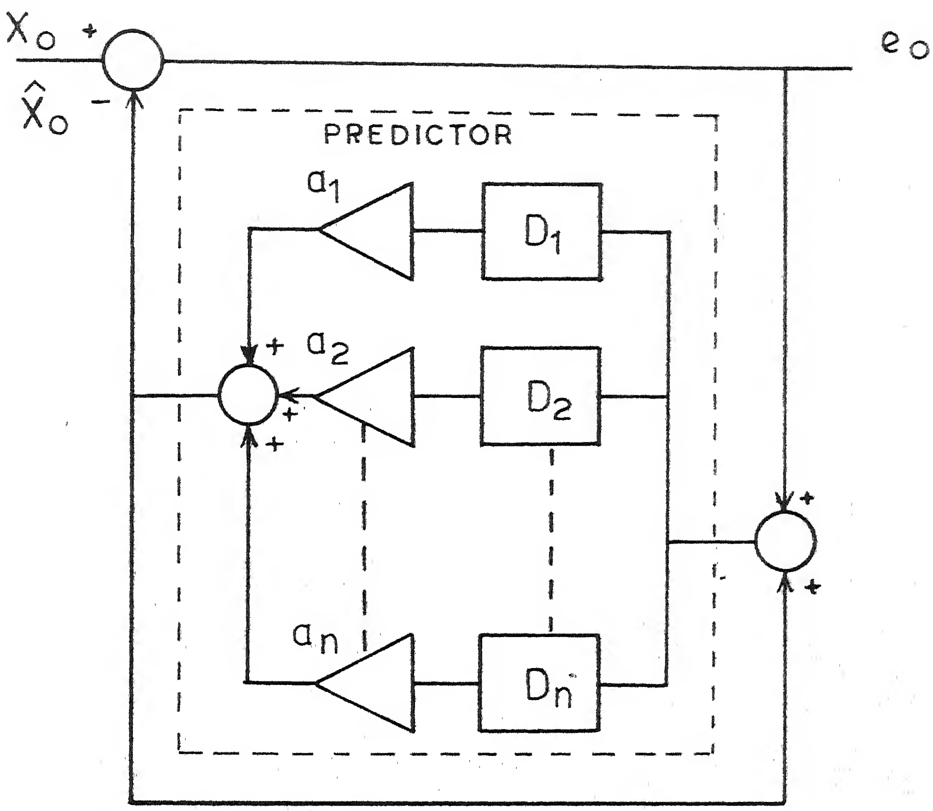
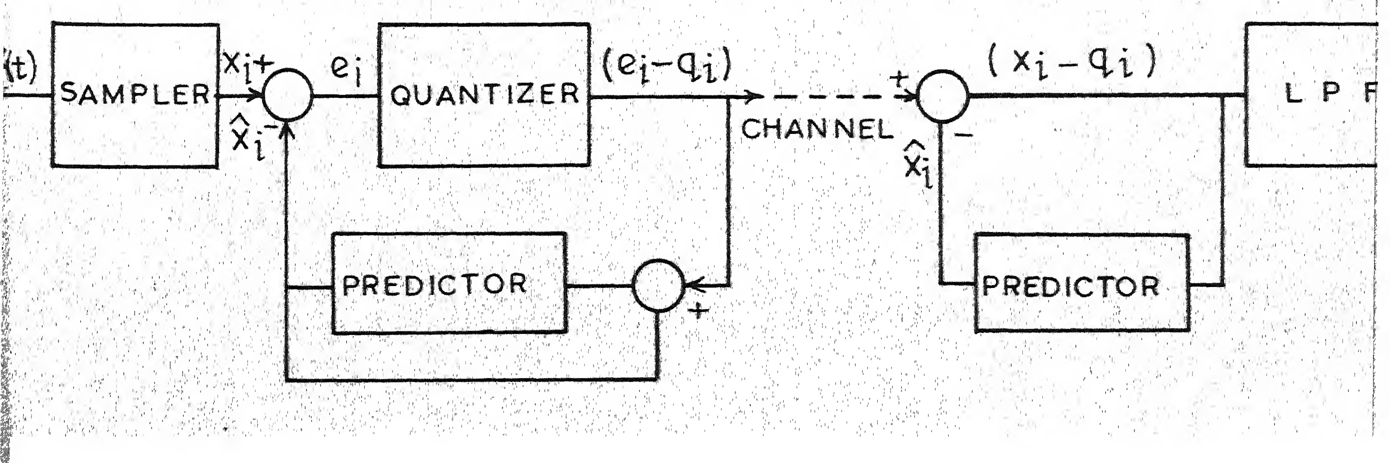


FIG. 3(a) BLOCK DIAGRAM OF A LINEAR PREDICTOR



CHAPTER 2

PSEUDO RANDOM NUMBERS AND TYPES OF INPUT SIGNALS

Discrete dynamic systems can either be deterministic or stochastic. Atleast one variable in a stochastic system is given by a probability function. There is thus a need of a source which can generate random variables so that such systems can be simulated. Though the method of generation is deterministic there is still a randomness about the numbers generated and as such they are called pseudo random numbers. The process being deterministic in itself constitutes an advantage as we can use the same set of numbers under different conditions while analyzing systems.

These random numbers are of immense use in the simulation of communication systems, as the input to such systems can be represented by a probability function. The system study undertaken needs three different types of input signals, viz. Gaussian with uniform power spectral density, Gaussian with an RC-spectrum and a Gauss-Markov process.

Gaussian signals with uniform power spectral density are used extensively for the simulation of data and FDM signals. A signal having an RC-spectrum is characterized with a constant spectral density upto a certain point and then it rolls of having a constant negative slope. Such

signals are used to simulate speech and television signals. It is, however, felt that speech signals are better simulated using a Gauss-Markov process. Each sample in such a signal has a Gaussian distribution but the process is Markov in so far as dependency of each sample is concerned upon the previous samples.

2. GENERATION OF INPUT SIGNALS

The basic source for generating all the desired input signals is a source generating uniformly distributed random numbers. Existing library subroutines have been modified and used to generate uniformly distributed random variables in the interval (0,1). The feature of the subroutines used is that the seed for generating the uniform variates is not a part of the subroutine but is external to it. This brings about some amount of flexibility and the subroutine can be initialized to conduct more than one set of simulations at any one time. The constant a_0 also called the multiplier and the seed x_0 for five different sets of uniformly distributed random variables are given below.

a_0	189277	186285	186293	186301	186309
x_0	11750920161	27594084237	10796931217	17669802561	560227513

2.1 Generation of Flat Band Limited Gaussian Signals

Gaussian variates with uniform power spectral density are obtained by using the Box Mueller Transformation given as

under :

$$S = SQRT(-2 \log_e RN1) \times Cos(2 \times \pi \times RN2), \qquad (2.1)$$

where RN1 and RN2 are two uniform random numbers in the range (0,1), S is the standard normal distribution. For a sample X from any normal distribution with mean μ and variance σ^2 we have

$$X = \sigma S + \mu \qquad (2.2)$$

2.2 Generation of RC-Shaped Gaussian Signals

For generating a RC-spectrum the flat Gaussian signal, band limited to the frequency $(0, f_o)$ is passed through a low-pass RC filter. The one sided power spectrum of the output is [4]

$$S_{xx}(f) = \frac{2\pi\alpha}{\tan^{-1}(2\pi f_o/\alpha)} \times \frac{1}{(2\pi f)^2 + \alpha^2}, \quad 0 < f < f_o \qquad (2.3)$$

where we assume a mean square value of unity and α equal to 1/RC. This RC- spectrum is relatively flat from f = 0 to a corner frequency of $f_c = (\alpha/2\pi)$ at which it rolls off to assume a slope of -6 dB per octave upto the cut off frequency of f_o.

2.3 Generation of Gauss-Markov Signals

Gauss-Markov can be of different order. This is dependent on the past history considered while generating

each sample. For this study Gauss-Markov process of the first order only has been considered. A schematic diagram to generate such a signal is given in Fig. 4.

Mathematically we can represent the generation as

$$X_r = C\,X_{r-1} + S_r \;; \quad C \dashrightarrow 1 \tag{2.4}$$

where X_r is a Gaussian random variable with zero mean and variance σ_x^2 and an adjacent sample correlation of C close to 1 (say $C \geqslant 0.9$). S_r is the input Gaussian process with zero mean and variance σ_s^2 such that [6]

$$\sigma_s^2 = \sigma_x^2\,(1 - C^2) \tag{2.5}$$

3. SIMULATION OF DIGITAL FILTERS

The Delta Modulation systems simulated need a low pass filter so as to produce a discrete output from a discrete input. Digital filtering may be looked upon as a numerical algorithm representing a discrete system with both input and output described by time series or number sequences. Thus if x_i and y_i denote the input and output signals respectively, then the filter algorithm takes the explicit form

$$y_i = a_i x_i + a_{i-1}\,x_{i-1} + \ldots\ldots + a_{i-n}\,x_{i-n} - b_1\,y_{i-1} + b_2\,y_{i-2} + \ldots\ldots + b_m\,y_{i-m}\,, \tag{2.6}$$

where the index i may run from $-\infty$ to ∞ or from 0 to ∞

depending upon whether the signals are one sided or two sided and a's and b's are real numbers.

Consider a numerical signal 'f' its Z-transform $F(z^{-1})$ is defined by

$$F(z^{-1}) = \sum_{n=0}^{\infty} f_n \, z^{-n}$$

where z is a complex variable. The operation of taking Z-transform of a signal is denoted by

$$F(z^{-1}) = Z(f)$$

The Z-transform is a linear operation and as such

$$Z(a.f + b.g + c.h) = a.Z(f) + b.Z(h) + c.Z(g)$$

where a,b and c are real numbers.

Consider the signal 'f' and another signal k_f related to 'f' as follows

$$k_{f_i} = 0 \qquad \text{for } i < k$$

$$k_{f_i} = f_{i-k} \qquad \text{for } i \geq k,$$

k_f is called the shifted version of 'f'. The Z-transform of such a shifted signal is given by

$$Z(k_f) = z^{-k} \, Z(f)$$

The product of Z-transforms

$$F(z^{-1}) = G(z^{-1})\, H(z^{-1})$$

is represented in the time domain as the discrete convolution

$$f_n = \sum_{j=0}^{n} g_j\, h_{n-j} \tag{2.7}$$

Using standard results of functional analysis it can be shown that the input and output of a linear filter are related through a convolution

$$y_i = \sum_{r=0}^{i} x_r\, h_{i-r} \tag{2.8}$$

where the sequence 'h' is independent of x and y and is called the impulse response. Taking Z-transform of both sides of (2.8) we have

$$Y(z^{-1}) = H(z^{-1})\, X\,(z^{-1}) \tag{2.9}$$

where $X(z^{-1})$, $Y(z^{-1})$ and $H(z^{-1})$ are the Z-transforms of the input signal, output signal and transfer function of the filter characterized by (2.8) and (2.9). The convolution in (2.7) can be expressed as a recurssive formula which would yield the result given in (2.6).

Standard Z-transform method is used to obtain the filter algorithm. The numerical low-pass filter used for the simulation studies is available as a programme package. The parameters are as follows :

OMS   -   Sampling frequency

OM1	-	Lower cut off frequency
TR	-	Transition ratio = $\frac{\text{lower cut off frequency}}{\text{upper cut off frequency}}$
R1	-	Pass band ripple
R2	-	Stop band ripple

4. SIMULATION RESULTS

Throughout the study 12,000 samples have been considered. Beyond this the samples no longer remain random. One set of Gaussian variates generated and used for the study were analyzed for mean, variance and correlation. The results are as under :

$$\text{Mean} = \frac{1}{N}\sum_{i=1}^{N} x_i = 0.023$$

$$\text{Variance} = \frac{1}{N}\sum_{i=1}^{N} x_i^2 = 1.029$$

$$\text{Correlation coefficients} = \frac{1}{N-m}\sum_{i=1}^{N} x_i . x_{i+m},$$

where m is an integer with which correlation is to be determined. The results for correlation upto the 10th sample are as under

m	1	2	3	4	5	6	7	8	9	10
r_m	0.013	-.008	0.00	0.05	0.017	0.001	0.009	0.006	0.010	0.001

The Gaussian variates are passed through a low-pass digital filter so as to obtain a band limited process. The sampling frequency (OMS) is fixed at 2 and the cut off frequency (OM1) is varied to obtain various sampling frequency to cut off frequency ratios. The pass band power for different cut off frequencies is measured. The parameters of the filter are

TR = 0.98 R1 = 0.05 R2 = 0.01 OMS = 2

and the power measured is indicated in the table below.

OM1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Power in pass band	0.099	0.197	0.293	0.397	0.498	0.597	0.689	0.789	0.873	0.929

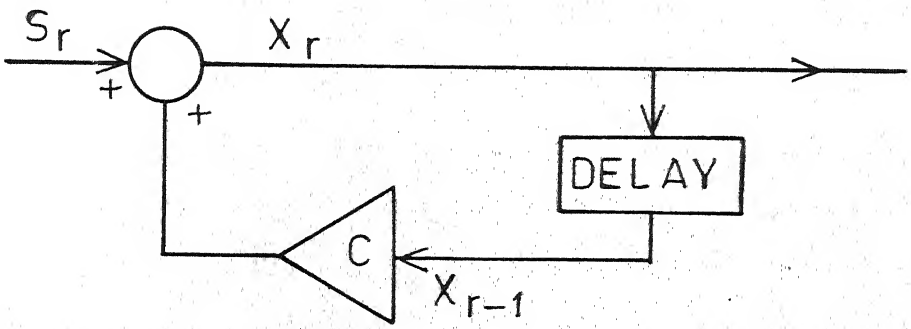


FIG. 4. GAUSS MARKOV GENERATION

CHAPTER 3

SIMULATION OF A DELTA MODULATION SYSTEM

In the simulation of the Delta Modulation system we have assumed that there is no over loading and no additive channel noise. The only noise in the system is the quantization noise. The input message signal has a normalized maximum frequency of unity and it is a zero mean process with variance equal to unity. Before proceeding with the simulation we have to design the step size. This has been done by Goodman [3] for a continuous process x(t). The no overload condition is expressed as the probability of slope of the input signal exceeding the maximum slope of the reconstructed signal being very small.

Let $S_{xx}(f)$ be the power spectral density of the input process x(t). The mean square value of the signal is given by

$$\sigma x^2 = \int_{-\infty}^{\infty} S_{xx}(f)\, df \tag{3.1}$$

and the effective band width is given by

$$f_e = \left[\frac{\int_{-\infty}^{\infty} f^2 S_{xx}(f)\, df}{\int_{-\infty}^{\infty} S_{xx}(f)\, df} \right]^{\frac{1}{2}} \tag{3.2}$$

The root mean square slope of x(t) is $2\pi f_e \sigma_x$ and maximum slope of the reconstructed signal is kfs where k

is the step size of the delta modulation system. It has been shown that if

$$kfs = 2\pi f_e \sigma_x c \tag{3.3}$$

the probability of over load for a value of $c = 4$ is only 6×10^{-5}.

The delta modulation system that has been simulated on the IBM 7044 system is depicted in Fig. 5. Pseudo random sequences are obtained according to the details in Section 2 of Chapter 2. These sequences are thereafter filtered through a low pass digital filter whose design and parameter details are given in Section 3 of Chapter 2. The various sampling frequencies for the system are simulated by altering the cut off frequency of the digital filter. Consider the sampling frequency to be M times the Nyquist rate. This is simulated by fixing the cut off frequency of the digital filter to 1/M. Thus the cut off frequency, of the digital filter at Nyquist rate is unity. The adder and delay element in the feedback loop around the quantizer constitutes an accumulator. The transfer function of the feedback loop is D/1-D where D is the unit delay operator e^{-ST} and represents a delay of one sample interval T. Since

$$\frac{D}{1-D} = D + D^2 + D^3 + \ldots\ldots$$

the signal being fedback is simply the sum of all previously transmitted samples. It is also identical to the decoded signal of the receiver and represents the signal value predicted for the next sample time. The difference between the input signal value x_i and this predicted signal y_{i-1} is quantized by a two level quantizer and transmitted through a discrete channel.

The noise n_i is obtained after low pass filtering the difference between x_i and y_i. This constitutes the quantization noise.

The effective band width (f_e) for an RC-spectrum input signal has been worked out by substituting (2.3) in (3.2) and taking the limits. The simplified expression is

$$f_e = \frac{\left[A\left(\frac{f_o}{\tan^{-1}(f_o/A)} - A\right) \right]^{\frac{1}{2}}}{\sigma_x}, \quad \text{where} \quad A = \alpha/2\pi$$

In carrying out the simulation using the RC-spectrum as the input the system remains essentially the same as in Fig. 5 with only an RC network following the low pass digital filter. The output of the digital filter is normalised so as to present a unity variance at the input of the RC network.

The mean square signal to mean square noise ratio for the system is given by

$$SNR(dB) = 10 \log_{10} \frac{\frac{1}{N}\sum_{i=1}^{N} x_i^2}{\frac{1}{N}\sum_{i=1}^{N} n_i^2}$$, where N is the number of samples taken for the simulation.

The various studies conducted using this delta modulation model with both the Gaussian and RC-spectrum inputs are as follows :

(a) Variation of signal to noise ratio with the variation of sampling frequency

(b) Variation of signal to noise ratio with variation of input variance

(c) Variation of signal to noise ratio with the variation of step size of the delta modulator.

2. SIMULATION OF PREDICTIVE QUANTIZATION SYSTEM

The system simulated is depicted in Fig. 3(b) with the predictor in the feedback loop being the same as in Fig. 3(a). The input is an RC-spectrum signal. The gains given by the matrix (1.18) require prior knowledge of correlation coefficients. These have been determined at various sampling frequencies as also with different samples. Detailed values obtained are in Table 1. Thereafter the gains a_i, $i = 1,2, \ldots\ldots, 10$ have been calculated. These are given in Table 2 for different sampling rates for i=1,2 and 3.

The difference between input and output signals, assuming an errorless transmission of digits is the quantization

SF/M	1	2	3	4	5
2	0.88617181	0.78380060	0.69561201	0.61837169	0.55203526
4	0.97029400	0.90354672	0.84086397	0.79570761	0.75417227
8	0.99359221	0.97380032	0.94523875	0.91293104	0.88134523
16	0.99869221	0.99392824	0.98624202	0.97602578	0.96378019
32	0.99959394	0.99856295	0.99682868	0.99441312	0.99134760
64	1.00009738	0.99990770	0.99956909	0.99908365	0.99845111

SF/M	6	7	8	9	10
2	0.49046009	0.43647905	0.38719029	0.34253737	0.30573286
4	0.70770171	0.66403474	0.62860159	0.59530659	0.55960187
8	0.85314724	0.82879478	0.80703569	0.78597314	0.76416609
16	0.95006610	0.93546224	0.92051557	0.90569908	0.89138559
32	0.98767132	0.98342961	0.97867535	0.97346402	0.96785601
64	0.99767601	0.99675887	0.99570365	0.99451429	0.99319233

Table 1

SF	a_1	a_2	a_3
2	0.88617318		
	0.89236897	-0.00699164	
	0.89244611	-0.01683643	0.01103219
4	0.97029458		
	1.59901036	-0.64796383	
	2.07224902	-1.81579672	0.73034723
8	0.99359334		
	2.03806298	-1.05120441	
	-1.91195510	6.60704368	-3.75761012
16	0.99869256		
	2.32033654	-1.32337427	
	1.21657299	0.61190563	-0.83404991
32	0.99959456		
	1.77105940	-0.77177778	
	1.25021278	0.42344068	-0.67485664
64	1.00010220		
	-0.42857350	1.42852972	
	3.37339276	0.28777871	-2.66133180

Table 2

error. Since the digital transmission rate of any system is finite, one has to use a quantizer which sorts the input into a finite number of ranges, N. For a given N, the system is described by specifying the end points x_k of the N input ranges, and an output level y_k corresponding to each input range. One would like to choose N y_ks for the corresponding x_ks such that the distortion D is minimum. It has been shown by Max [8] that $D = kN^{-2}$ where k is a constant.

At the outset a single predictor has been used in the feedback path and using a two level quantizer. The results obtained are not much different to those of an LDM system. This is because the gain a_1 is almost unity and as such the predictor is nothing but a leaky integrator. Large number of predictors in the feedback path using a two level quantizer give rise to instability in the system.

Thereafter, the quantizer was progressively replaced by a 4,8 and 16 level quantizer and the signal to noise ratio measured. Increasing the predictive path to 2 and thereafter to 3 gave no appreciable change in the results. Values of x_ks and y_ks used are given in Table 3.

Another predictive system simulated uses the Gauss-Markov input. The system diagram is given in Fig. 6. The output of the Delta Modulator for a Gauss-Markov input as given by (2.4) is depicted by

$$b_r = \mathrm{Sgn}\ (X_r - aY_{r-1})$$

Level	x_k	y_k
4	0.0	0.4528
	0.9816	1.510
8	0.0	0.2541
	0.5006	0.7560
	1.050	1.344
	1.748	2.152
16	0.0	0.1284
	0.2582	0.3881
	0.5224	0.6568
	0.7996	0.9424
	1.099	1.256
	1.437	1.618
	1.844	2.069
	2.401	2.733

Table 3

$$K = K \; b_r$$

$$Y_r = a.Y_{r-1} + K \tag{3.3}$$

where b_r is the DM-bit, 'a' is the predictive path gain and K the step length. Using a one bit quantizer we have the difference signal Q_r given by

$$Q_r = X_r - a.Y_{r-1} \tag{3.4}$$

The quantization error E_r is the difference between the output and the input

$$E_r = Y_r - X_r \tag{3.5}$$

Using (2.4), (3.4) and (3.5) the quantizer input can be written as

$$\begin{aligned} Q_r &= (c\, X_{r-1} + S_r) - a(X_{r-1} + E_{r-1}) \\ &= S_r + (c - a)\, X_{r-1} - a\, E_{r-1} \end{aligned} \tag{3.6}$$

If K_{opt} represents the optimum step length, σ_q^2 the variance of the quantizer input and σ_e^2 the variance which gives the minimum quantization error we have [6]

$$\begin{aligned} K_{opt} &= L.\sigma_q \\ \sigma_e^2 &= \sigma_q^2/M \end{aligned} \tag{3.7}$$

Assuming the quantizer input, Q_r as also the input signal, X_r to have a Gaussian distribution we have

$$L = \sqrt{2/\pi} \approx 0.80$$

$$M = (1 - 2/\pi)^{-1} \approx 2.78$$

The variance of Q_r can be approximated by squaring (3.6), and taking the expected value, neglecting the contribution due to cross products we thus get variance of Q_r as a sum of squares :

$$\sigma_q^2 = \sigma_s^2 + (c - a)^2 \sigma_x^2 + a^2\sigma_e^2 \tag{3.8}$$

From (2.5), (3.7) we can rewrite (3.8) as under :

$$\frac{\sigma_x^2}{\sigma_q^2} = \frac{1 - a^2/M}{1 - 2ac + a^2} \tag{3.9}$$

To determine the optimum gain 'a' we take derivative of (3.9) and equated it to zero. The optimal predictor gain a_{opt} is given as under :

$$a_{opt} = \left[\frac{1 + M}{2c}\right] - \left[\left(\frac{1 + M}{2c}\right)^2 - M\right]^{1/2} \tag{3.10}$$

To begin with the gain 'a' of the predictive path is kept constant and equal to 'c' and the signal to noise ratio measured. Thereafter the predictive path gain is varied through the optimal predictor gain as calculated by (3.10) and the signal to noise ratio measured till the system becomes unstable.

3. DATA COMPRESSION AND SIMULATION OF AN ADAPTIVE DELTA MODULATION SYSTEM

A Linear Delta Modulation system using an RC-spectrum as the input signal is simulated at a sampling frequency of 16. The output bit pattern emanating from the quantiser is considered as a stream of 4 bit words. Their distribution gives an indication as to the redundancy or near redundancy of including some words in the data to be transmitted. Accordingly these are eliminated resulting in a reduction from 16 to 8 combinations, thereby reducing the word length of transmitted data from 4 to 3 bits. At the receive end the data was converted to 4 bits and the original message as transmitted was approximately reconstructed. The signal to noise ratio is again measured so as to have a qualitative assessment of the degradation in the system. The noise includes both quantization noise as also noise introduced due to data compression. The study presumes that no overloading occurs and the transmission channel losses are zero.

The above process using the same system set up was repeated but the system was made adaptive in so far as its step length was concerned using the algorithm as given in Section 4 of Chapter 1. Such a system requires a knowledge of the previous bit output of the quantizer. The step length is either increased or decreased by a factor M depending on

whether the present bit is the same or different as compared to the previous bit. This takes care of overload condition in the system, the step length being adaptive. The optimum value of M as suggested by Jayant [6] is 1.1. Once again the signal to noise ratio of the system is determined.

4. SIMULATION RESULTS

(a) The input is first a Gaussian band limited signal with an uniform spectral density uniform over the band (0,1). The next input has an RC-spectrum. For various sampling frequencies the signal to noise ratio is determined. The results are tabulated below.

Input/SF	2	4	8	16	32	64
Uniform SNR(dB)	-13.23	-1.28	8.97	18.26	26.91	34.76
RC-spectrum SNR(dB)	-0.65	10.88	20.51	29.11	38.64	48.16

It is evident that the signal to noise ratio in case of the RC-spectrum is higher than in the case of the uniform spectrum signal input, however, the 9 dB increase per octave, as shown by O'Neal is maintained. These results have been represented on a graph in Fig. 7.

(b) The sampling frequency is fixed at 8 and the input variance is varied from -20 dB to +20 dB. The signal to noise ratios as measured are given below.

Input variance(dB)	-20.00	-14.77	-10.00	-4.77	0
Uniform SNR (dB)	-17.31	-11.26	-5.1	2.63	8.97
RC-spectrum SNR(dB)	-4.18	2.77	8.39	14.67	20.51
Input variance	+4.77	10.00	14.77	20.00	
Uniform SNR(dB)	12.64	9.29	4.86	2.35	
RC-spectrum SNR(dB)	24.00	17.97	10.74	6.29	

These results have been plotted in Fig. 8.

(c) The sampling frequency and other parameters are kept fixed and the step length varied. The signal to noise ratio for a sampling frequency of 32 are given below.

Step Length	K/4	K/2	K	2K	4K	8K
Uniform SNR(dB)	7.58	21.57	26.92	19.68	10.99	1.71
RC-spectrum SNR(dB)	17.75	34.95	38.64	30.90	22.23	13.78

The results for a uniform input have been shown plotted in Fig. 9 and those for an RC-spectrum in Fig. 10.

(d) For a predictive delta modulation system the signal to noise ratio are measured with one stage of prediction using an RC-spectrum input. The results are given below.

SF	2	4	8	16	32	64
SNR(dB)	-.84	11.07	20.53	29.49	38.36	47.24

It will be observed that the above results are marginally different to those obtained using a delta modulator. This is because such a system has a predictive path gain almost equal to unity and as such it behaves like a delta modulator having a leaky integrator in its feed back path.

The results of a predictive system using higher levels of quantization are as under

SF/Levels	4	8	16
2 SNR(dB)	11.65	17.14	22.72
4 SNR(dB)	15.59	20.06	25.73

(e) The signal to noise ratio for a predictive system having a Gauss-Markov input gives the following signal to noise ratio.

SF	4	8	16
SNR(dB)	14.91	23.38	31.96

These results are plotted in Fig. 7. The predictive path gain has been kept equal to the gain used for generating the Gauss-Markov process.

The signal to noise variations versus the predictive gain keeping the gain in the Gauss-Markov generation loop as constant are tabulated below.

Gain	0.0	0.1	0.2	0.3	0.4	0.5	0.6
SNR(dB)	5.11	5.37	5.80	6.17	6.58	7.05	7.69
Gain	0.7	0.8	0.9	0.92	0.93	0.94	0.95
SNR(dB)	8.77	11.18	18.28	21.13	21.84	22.15	22.96
Gain	0.96	0.97	0.98	0.99			
SNR(dB)	23.30	23.64	23.82	23.50			

These results have been plotted in Fig. 11.

(f) The distribution of 4 bit binary words at the output of

the output of the delta modulator using 8 different sets of random number generators is given in Table 4.

The signal to noise ration on data compression is 20.61 dB as against 29.11 dB without any compression. Words with a probability less than or equal to .004 have not been transmitted.

From Table 4 it is amply clear that the bit pattern for all the random number generators used is approximately the same. Hence only one random number generator was used for an Adaptive system. The distribution pattern was as under.

Word	0	1	2	3	4	5	6	7	8	9	10
Distr.	325	130	236	25	274	221	131	125	145	146	240
Word	11	12	13	14	15						
Distr.	236	25	253	129	358						

As the distribution was found to be more or less uniform no attempt has been made to compress the data.

Total number of words = 3000

Word	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	11	29	220	13	259	763	204	24	17	209	723	241	10	236	35	6
	14	38	259	10	247	703	223	42	36	231	652	223	10	237	48	27
	4	33	223	1	250	779	233	22	31	215	685	237	7	249	24	7
	10	33	262	7	253	670	210	36	28	238	703	225	6	256	46	17
	11	33	237	4	256	675	231	28	30	213	730	237	6	267	36	6
	9	32	262	9	257	681	223	21	32	220	683	231	6	279	41	14
	3	28	255	4	252	684	216	27	28	221	739	249	5	247	38	4
	7	42	241	11	261	720	207	43	25	229	672	264	8	236	24	10

Table 4

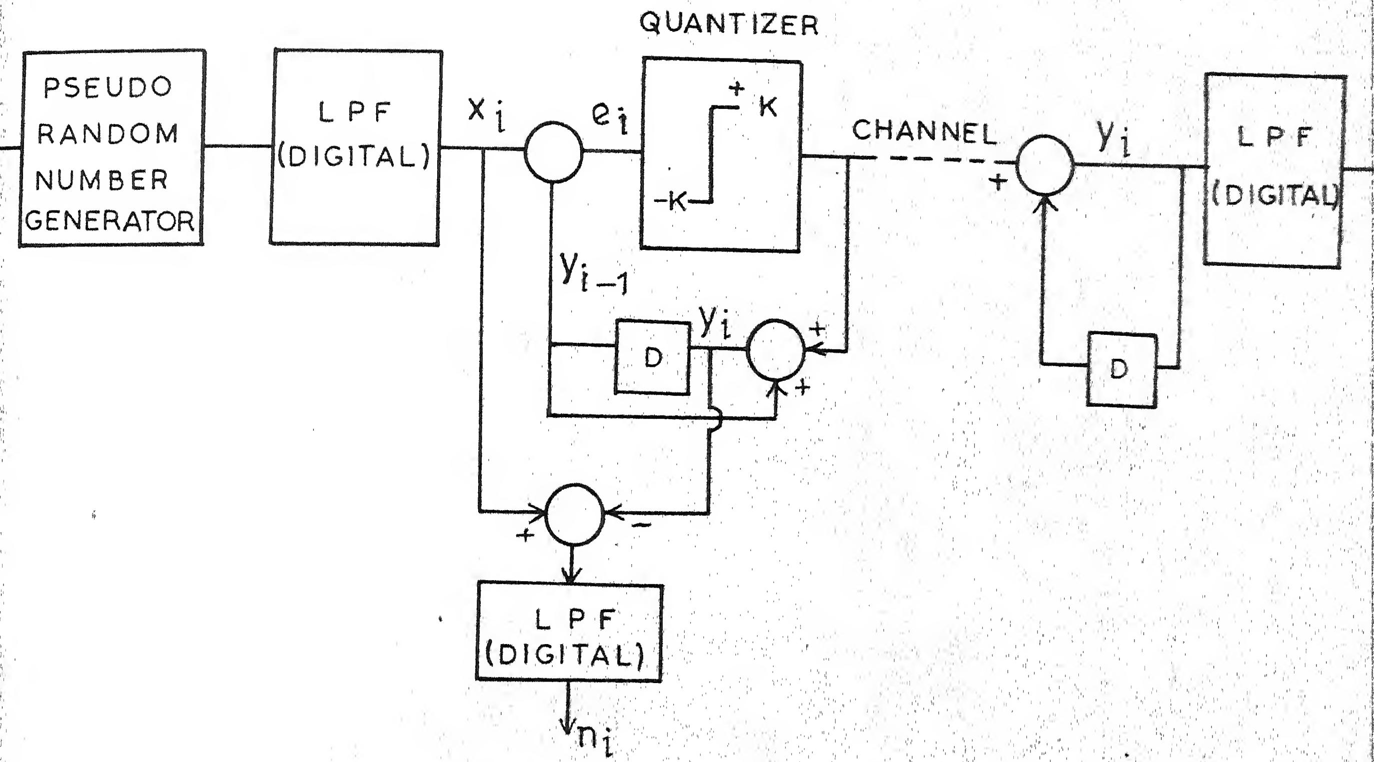


FIG. 5. DELTA MODULATION SYSTEM AS SIMULATED.

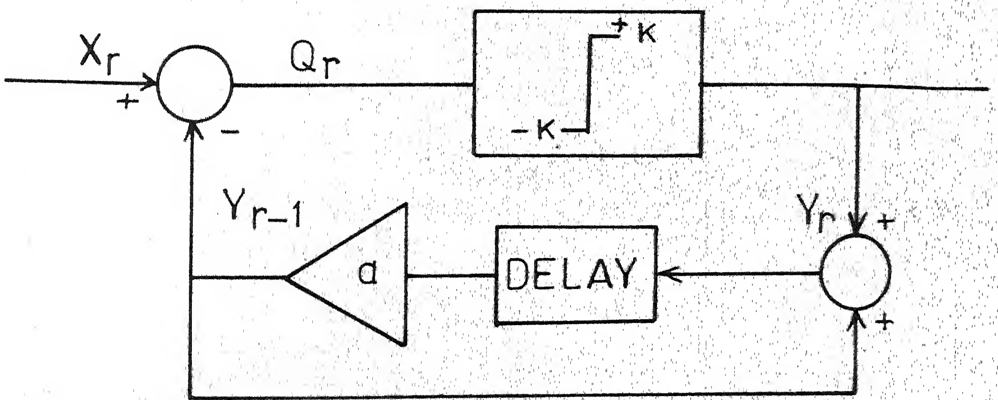


FIG. 6. DELTA MODULATION OF A FIRST ORDER GAUSS MARKOV PROCESS

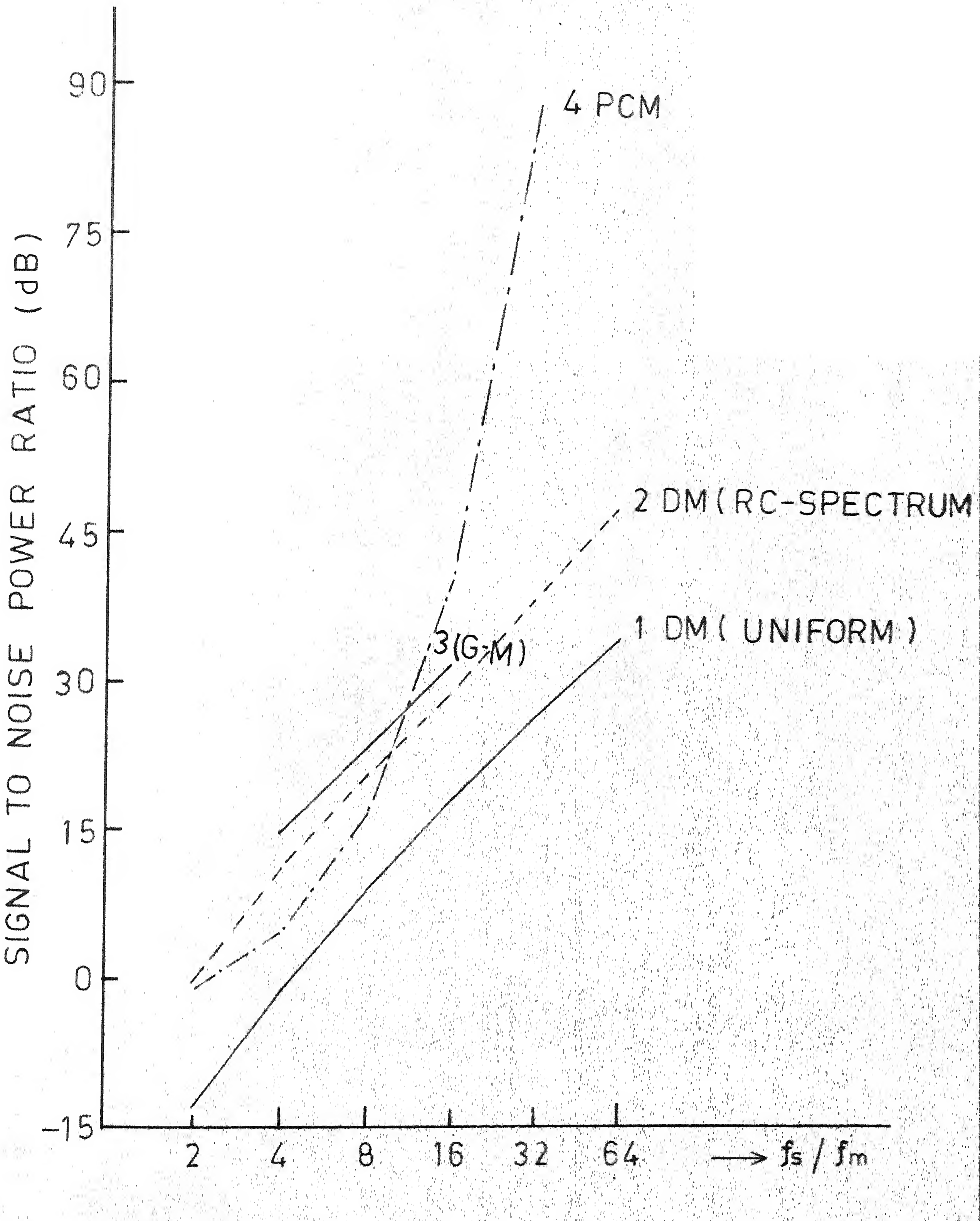


FIG.7. FREQUENCY VERSUS SIGNAL TO NOISE POWER RATIO

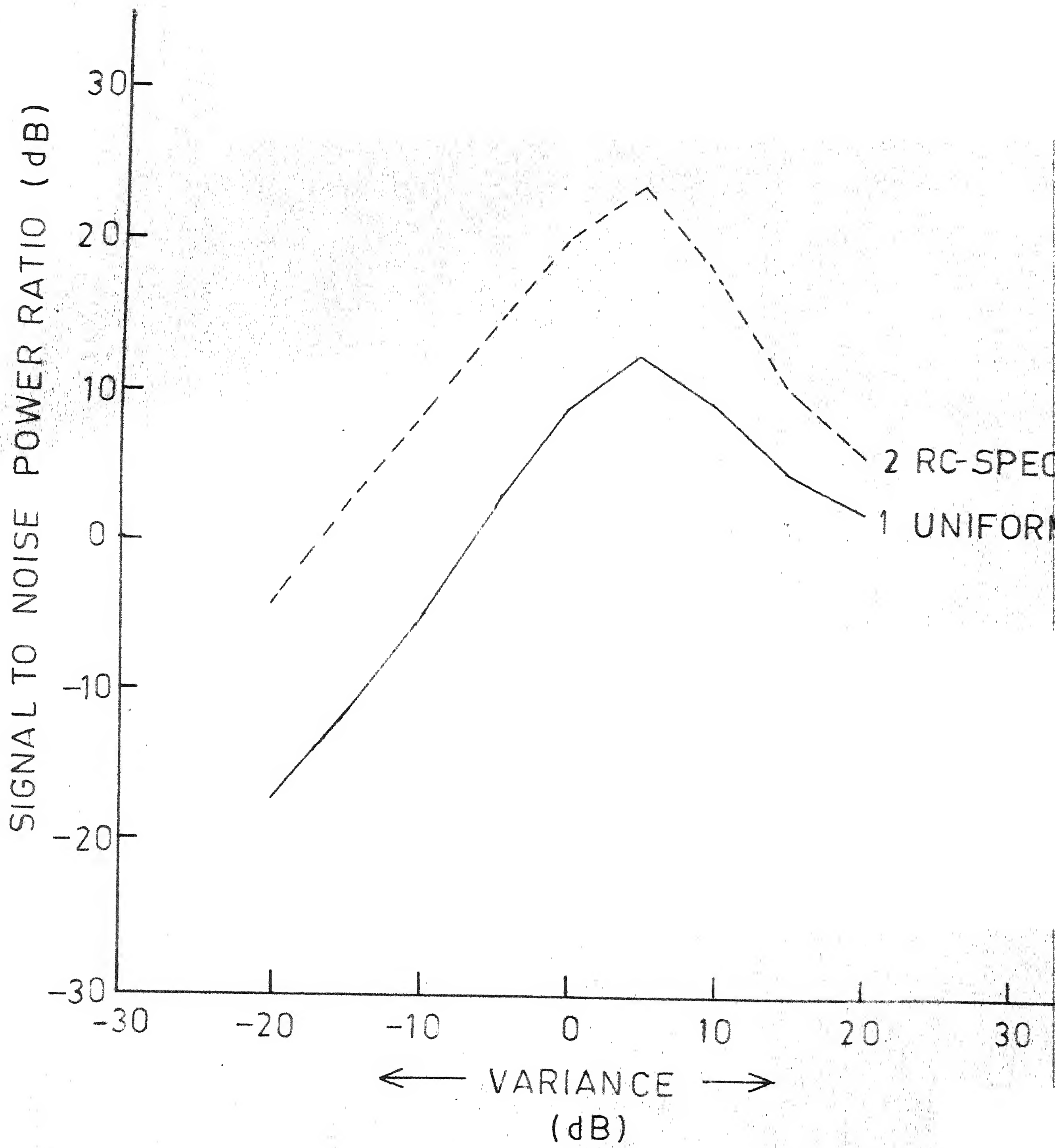


FIG.8. VARIANCE VERSUS SIGNAL TO NOISE POWER RATIO

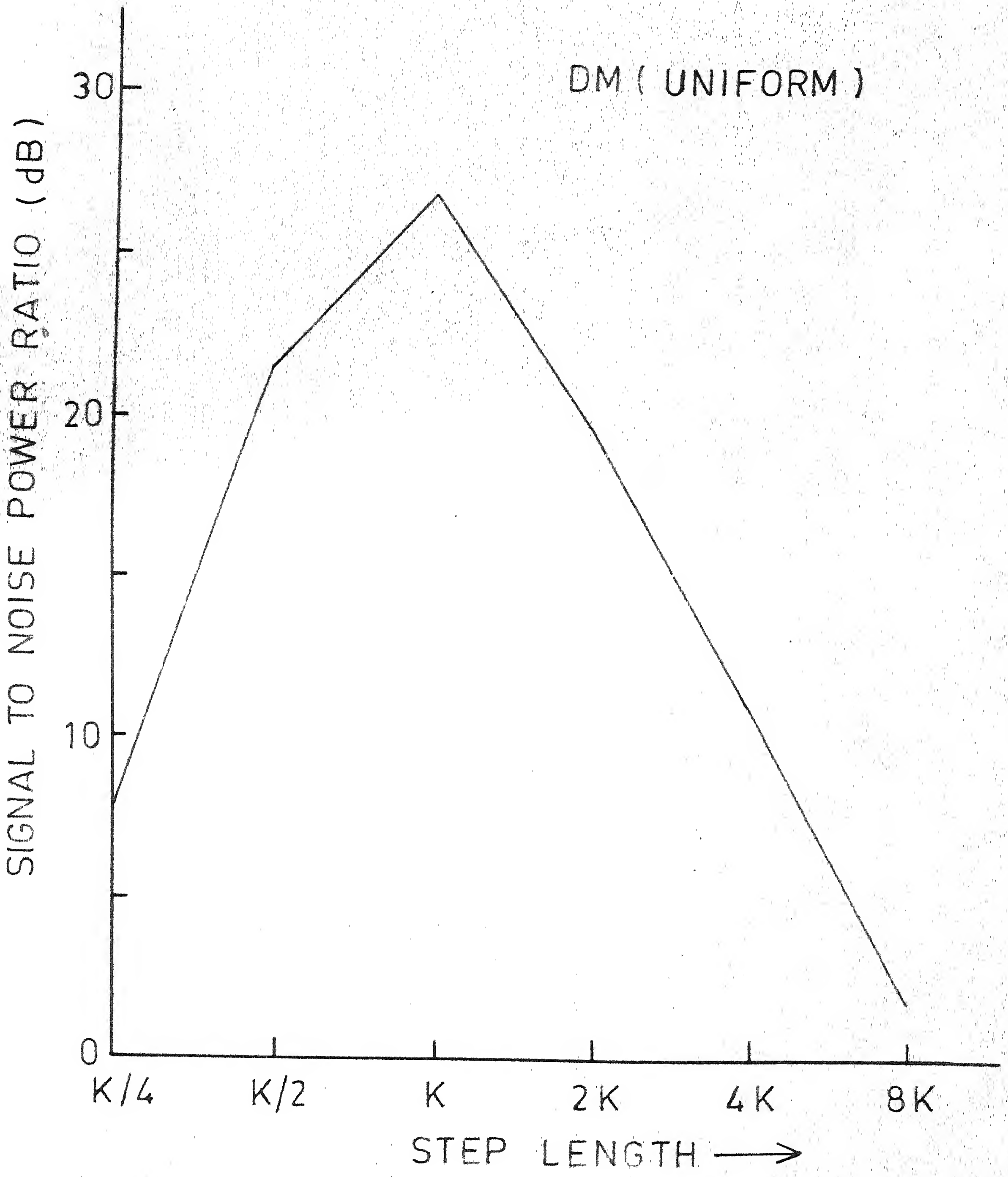


FIG.9. STEP LENGTH VERSUS SIGNAL TO NOISE POWER RATIO

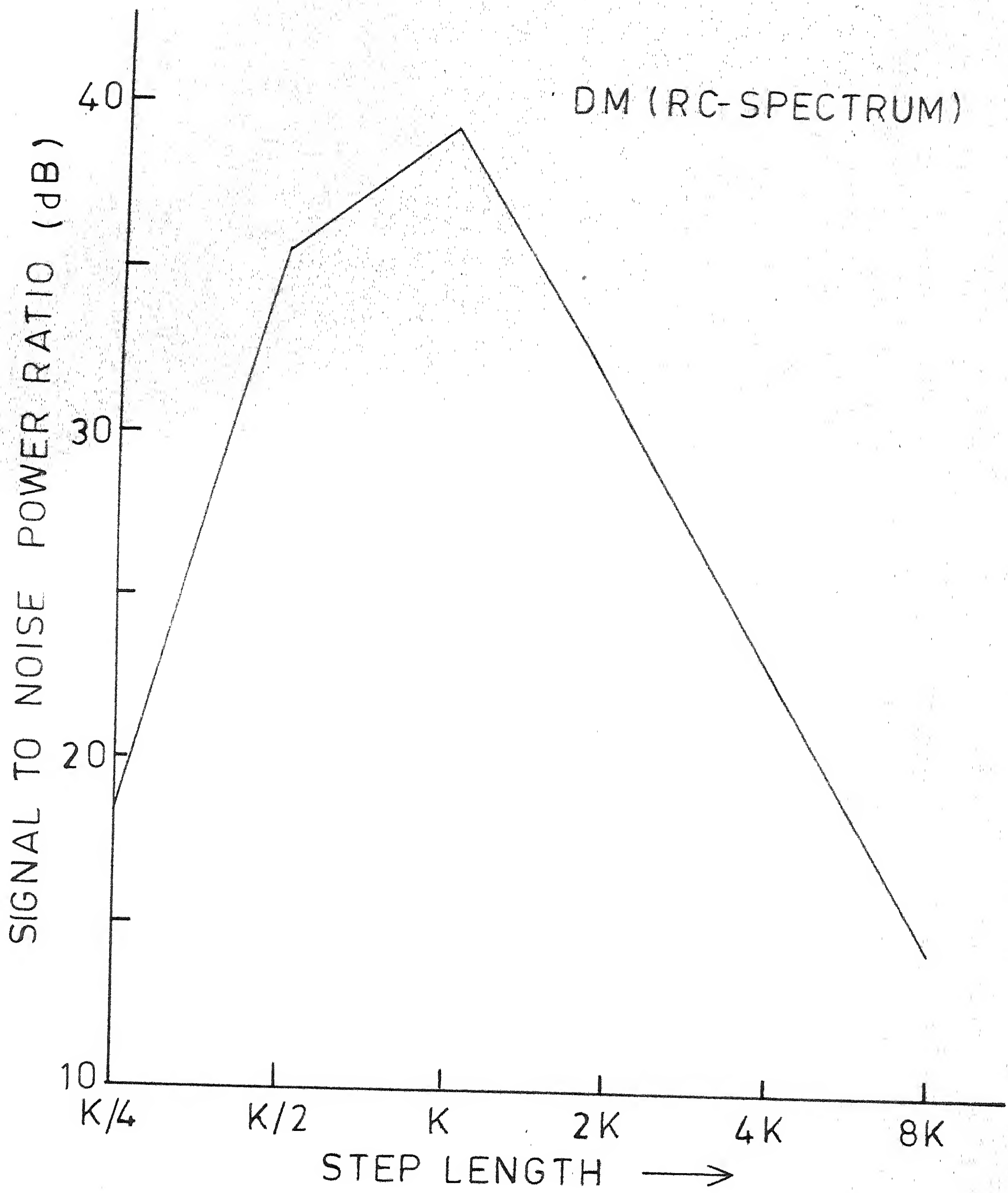


FIG.10. STEP LENGTH VERSUS SIGNAL TO NOISE POWER RATIO

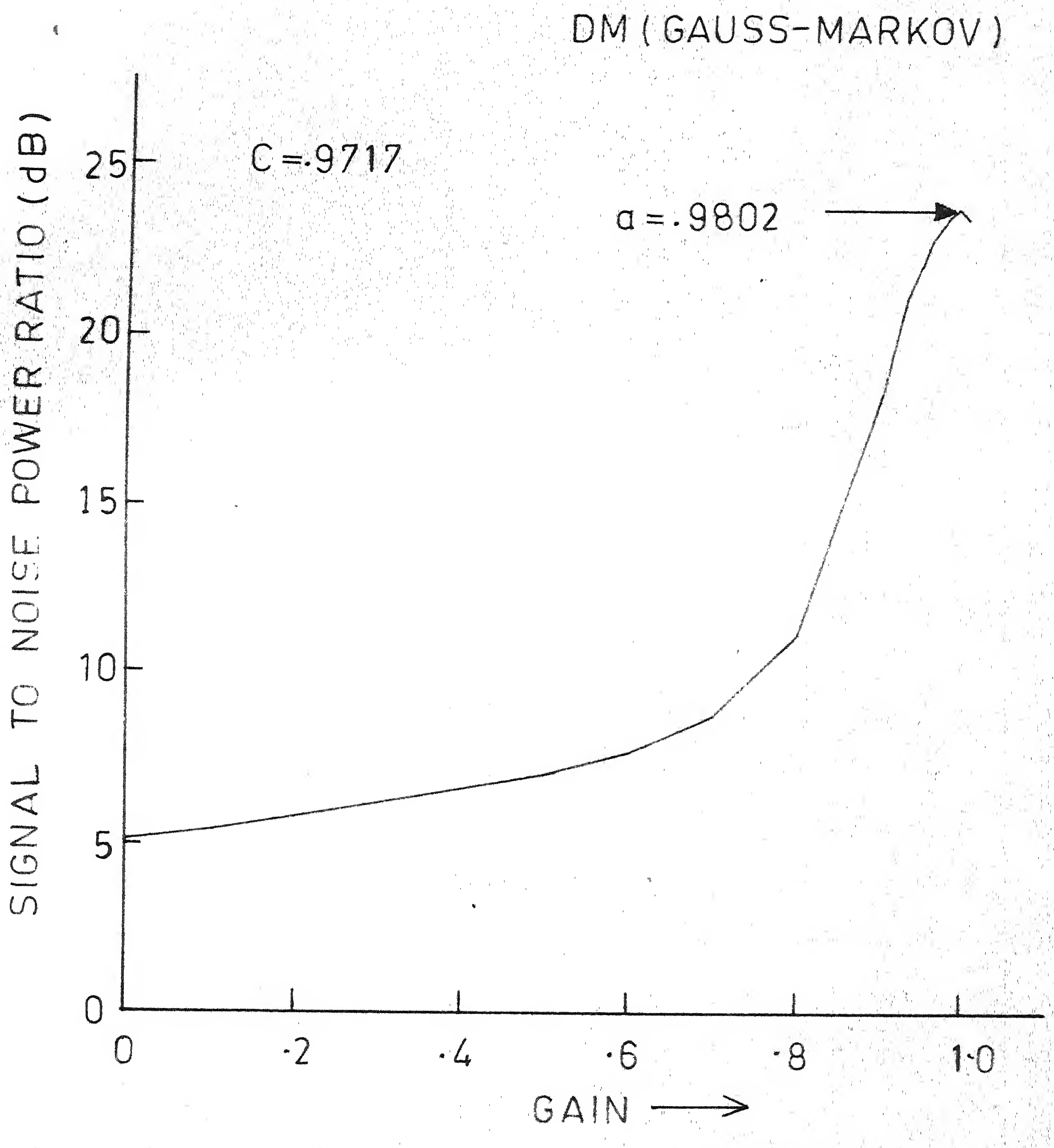


FIG.11. GAIN VERSUS SIGNAL TO NOISE POWER RATIO

CHAPTER 4

CONCLUSION

The simulation studies conducted have been aimed at studying Linear Delta Modulation and Predictive Quantization systems. The systems have been so designed that the noise at the receiver output is only due to quantization of the input. The transmission channel has been assumed to be noise free. The results obtained conform favourably with the results obtained by others.

The Linear Delta Modulation system has been studied with three different types of input signals, viz. a flat band-limited Gaussian signal, a RC-shaped Gaussian signal and a Gauss-Markov signal. Considering a sampling frequency of 16 the signal to noise ratios obtained are 18.26 dB, 29.11 dB and 31.96 dB respectively. The system performance as indicated by the signal to noise ratios is best for a Gauss-Markov signal input, followed by a RC-shaped Gaussian input and poorest for a flat Gaussian signal. The increase of 9 dB per octave in signal to noise ratio is, however, maintained in all three cases.

An interesting feature for Gauss-Markov signals is the manner in which optimum step length is calculated [6] and is different to the method suggested by Goodman [3]. The initial results assume the predictive path gain to be the

same as the loop gain of the Gauss-Markov generator. Later this path gain has been varied to show that there is an optimum value for which the signal to noise ratio is maximum.

The Predictive Quantization system studied use a single predictor in the feedback path. The system has been studied for 2,4,8 and 16 levels of quantization. Non uniform quantizers have been used in each case, and the quantizer characteristics were chosen [10] to give minimum mean-squared error in the output. A two level quantizer and a sampling frequency of 4 gave a signal to noise ratio of 10.88 dB whereas the same bit rate obtained by using a sampling frequency of 2 and a 4 level quantizer gave a signal to noise ratio of 11.65 dB. The results are almost comparable. On the other hand a 16 level quantizer with a sampling frequency of 2 gave a signal to noise ratio of 22.72 dB whereas choosing the same bit rate an 8 level quantizer with a sampling frequency of 4 gave a lower signal to noise ratio of 20.06 dB. Higher levels of prediction did not yield any increase in the signal to noise ratio.

Since both DM and DPCM methods are slope limited it is necessary that the behaviour of such systems using adaptive techniques should be studied in detail. Such techniques would ensure that the instantaneous slope variation of the input would be better reproduced in the output, thereby reducing the slope overload noise. By using an optimum

algorithm like the type suggested by Jayant [6], as indicated in Section 4 of Chapter 1, it may be possible to substantially reduce the overload noise.

An adaptive system does add to the complexity of the circuitry vis-a-vis a Linear Delta Modulation system but it does increase the dynamic range.

The results for bit rate reduction do indicate that a reduction in the bit rate is possible but this is not very realistic from the point of view of actual systems. There is a decrease of 9 dB in the signal to noise ratio in reducing the transmitted bits from 4 to 3. The same signal to noise ratio can be achieved by reducing the sampling rate to half but the ratio of bit rate reduction is $\frac{1}{2}$ as against $\frac{3}{4}$ in the case studied.

Finally, it must be said that real signals like speech and television can not truly be judged by the signal to noise power ratio measurement alone. A subjective evaluation is in fact more appropriate. It is possible to sample speech and code the sampled signal which is put on a magnetic tape; like wise pictures can be scanned and the scanned signal can be coded to generate a coded signal on magnetic tape. The magnetic tape so created is used as an input to the simulated Linear Delta Modulation or Predictive Quantization systems.

The output is obtained on a magnetic tape which is used to recreate the original signal which makes the subjective evaluation feasible.

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